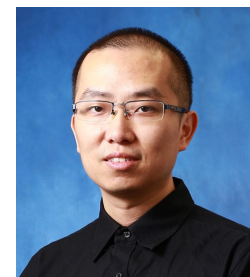
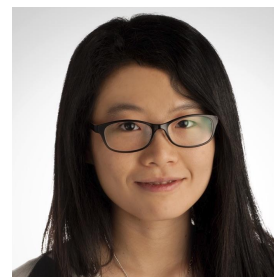




Rectified Diffusion: Straightness Is Not Your Need in Rectified Flow

Fu-Yun Wang¹ Ling Yang² Zhaoyang Huang¹ Mengdi Wang³ Hongsheng Li¹

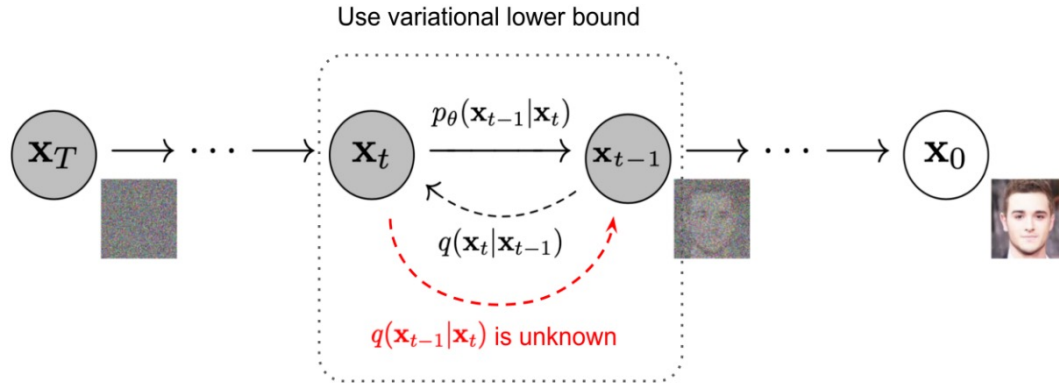
¹MMLab@CUHK ²Peking University ³Princeton University



<https://github.com/G-U-N/Rectified-Diffusion>



Diffusion Models: Markovian Perspective



Assumption:

- $$p(\mathbf{x}_{0:T}) = p(\mathbf{x}_T) \prod_{t=1}^T p_{\theta}(\mathbf{x}_{t-1}|\mathbf{x}_t)$$

Forward Process:

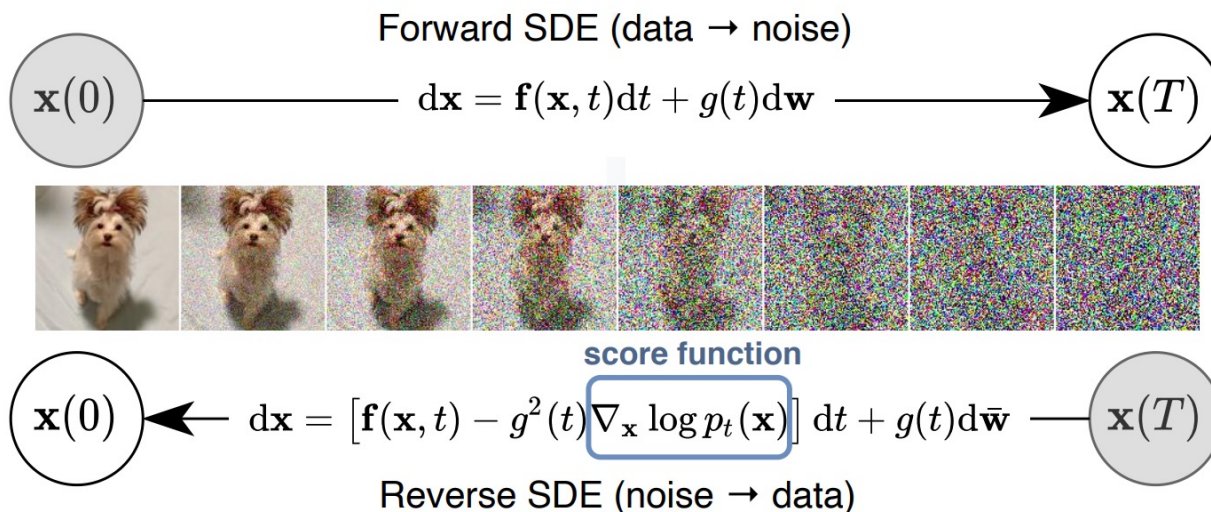
- $$q(\mathbf{x}_t|\mathbf{x}_{t-1}) = \mathcal{N}(\mathbf{x}_t; \sqrt{\alpha_t}\mathbf{x}_{t-1}, (1 - \alpha_t)\mathbf{I})$$
- $$p(\mathbf{x}_T) = \mathcal{N}(\mathbf{x}_T; \mathbf{0}, \mathbf{I})$$

Reverse Process:

- $$q(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{x}_0) = \frac{q(\mathbf{x}_{t-1}|\mathbf{x}_t, \mathbf{x}_0)q(\mathbf{x}_t|\mathbf{x}_0)}{q(\mathbf{x}_{t-1}|\mathbf{x}_0)}$$
- Maximum Likelihood Estimation (MLE)**
 is Equivalent to

$$\arg \min_{\theta} \mathbb{E}_{t \sim U\{2, T\}} [\mathbb{E}_{q(\mathbf{x}_t|\mathbf{x}_0)} [D_{\text{KL}}(q(\mathbf{x}_{t-1}|\mathbf{x}_t, \mathbf{x}_0) \parallel p_{\theta}(\mathbf{x}_{t-1}|\mathbf{x}_t))]]$$

Diffusion Models: Stochastic Differential Equation Perspective



Probability Flow ODE:

A deterministic reverse process

$$d\mathbf{x} = \left[\mathbf{f}(\mathbf{x}, t) - \frac{1}{2}g(t)^2 \nabla_{\mathbf{x}} \log p_t(\mathbf{x}) \right] dt$$

Exact Solution form of PF-ODE

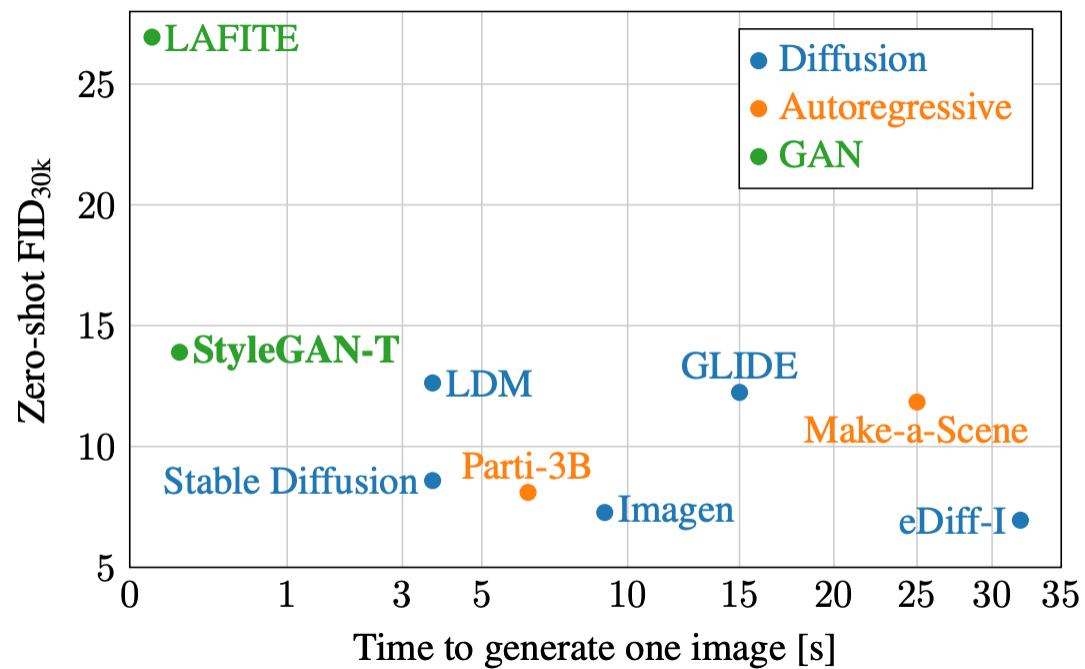
$$\mathbf{x}_t = \frac{\alpha_t}{\alpha_s} \mathbf{x}_s - \alpha_t \int_{\lambda_s}^{\lambda_t} e^{-\lambda} \hat{\epsilon}_{\theta}(\hat{\mathbf{x}}_{\lambda}, \lambda) d\lambda.$$

The only unknown term is the **score function**.

Train a neural network through score matching!



Diffusion Models: Slow Inference Speed



How to speed up the diffusion generation?

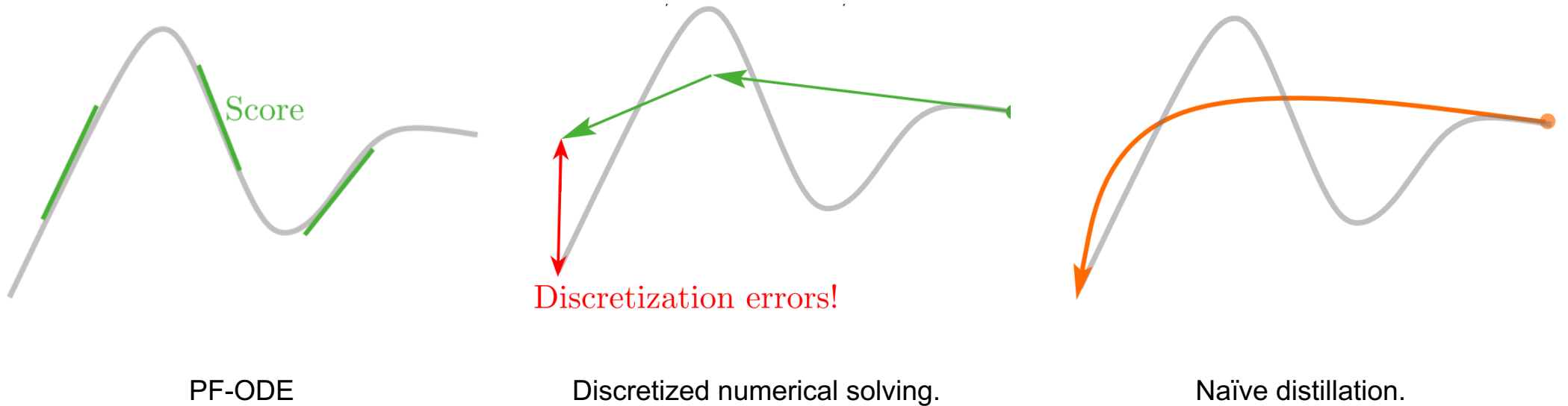
- Reducing the number of function evaluation (NFE).
- Better Solvers.
- Adversarial post-training.
- Distillation.
 - Naïve distillation.
 - Guided distillation.
 - Score distillation.
 - Consistency distillation.
 - Rectification.



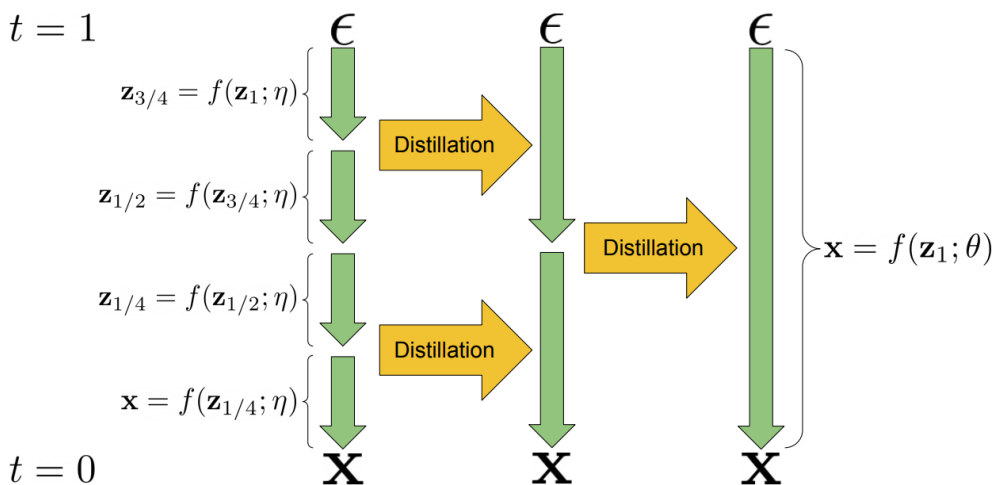
Understanding Diffusion Models from the PF-ODE path

We know the derivative w.r.t. time t .

$$d\mathbf{x} = \left[\mathbf{f}(\mathbf{x}, t) - \frac{1}{2}g(t)^2 \nabla_{\mathbf{x}} \log p_t(\mathbf{x}) \right] dt$$



Distillation Techniques: Progressive Distillation



Algorithm 2 Progressive distillation

Require: Trained teacher model $\hat{\mathbf{x}}_\eta(\mathbf{z}_t)$

Require: Data set \mathcal{D}

Require: Loss weight function $w()$

Require: Student sampling steps N

for K iterations **do**

$\theta \leftarrow \eta$ \triangleright Init student from teacher

while not converged **do**

$\mathbf{x} \sim \mathcal{D}$

$t = i/N, i \sim \text{Cat}[1, 2, \dots, N]$

$\epsilon \sim N(0, I)$

$\mathbf{z}_t = \alpha_t \mathbf{x} + \sigma_t \epsilon$

2 steps of DDIM with teacher

$t' = t - 0.5/N, t'' = t - 1/N$

$\mathbf{z}_{t'} = \alpha_{t'} \hat{\mathbf{x}}_\eta(\mathbf{z}_t) + \frac{\sigma_{t'}}{\sigma_t} (\mathbf{z}_t - \alpha_t \hat{\mathbf{x}}_\eta(\mathbf{z}_t))$

$\mathbf{z}_{t''} = \alpha_{t''} \hat{\mathbf{x}}_\eta(\mathbf{z}_{t'}) + \frac{\sigma_{t''}}{\sigma_{t'}} (\mathbf{z}_{t'} - \alpha_{t'} \hat{\mathbf{x}}_\eta(\mathbf{z}_{t'}))$

$\tilde{\mathbf{x}} = \frac{\mathbf{z}_{t''} - (\sigma_{t''}/\sigma_t) \mathbf{z}_t}{\alpha_{t''} - (\sigma_{t''}/\sigma_t) \alpha_t}$ \triangleright Teacher $\hat{\mathbf{x}}$ target

$\lambda_t = \log[\alpha_t^2 / \sigma_t^2]$

$L_\theta = w(\lambda_t) \|\tilde{\mathbf{x}} - \hat{\mathbf{x}}_\theta(\mathbf{z}_t)\|_2^2$

$\theta \leftarrow \theta - \gamma \nabla_\theta L_\theta$

end while

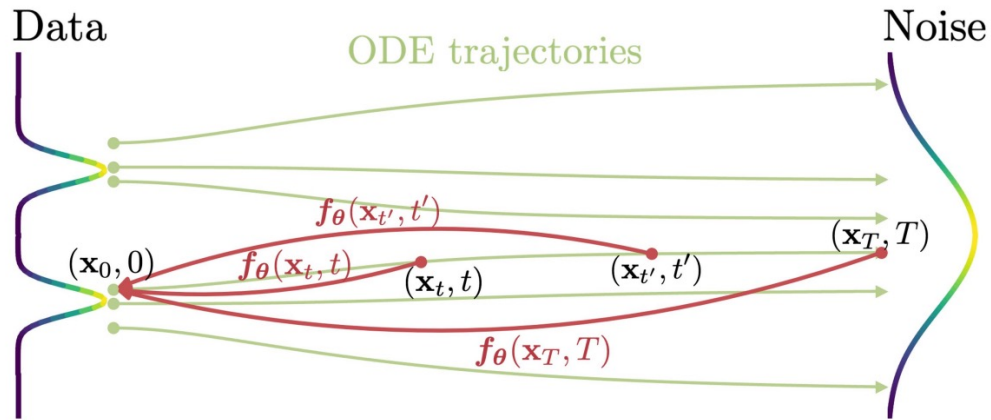
$\eta \leftarrow \theta$ \triangleright Student becomes next teacher

$N \leftarrow N/2$ \triangleright Halve number of sampling steps

end for



Distillation Techniques: Consistency Distillation



Algorithm 2 Consistency Distillation (CD)

Input: dataset \mathcal{D} , initial model parameter θ , learning rate η , ODE solver $\Phi(\cdot, \cdot; \phi)$, $d(\cdot, \cdot)$, $\lambda(\cdot)$, and μ

$\theta^- \leftarrow \theta$

repeat

 Sample $\mathbf{x} \sim \mathcal{D}$ and $n \sim \mathcal{U}[[1, N - 1]]$

 Sample $\mathbf{x}_{t_{n+1}} \sim \mathcal{N}(\mathbf{x}; t_{n+1}^2 \mathbf{I})$

$\hat{\mathbf{x}}_{t_n}^{\phi} \leftarrow \mathbf{x}_{t_{n+1}} + (t_n - t_{n+1})\Phi(\mathbf{x}_{t_{n+1}}, t_{n+1}; \phi)$

$\mathcal{L}(\theta, \theta^-; \phi) \leftarrow$

$\lambda(t_n)d(\mathbf{f}_{\theta}(\mathbf{x}_{t_{n+1}}, t_{n+1}), \mathbf{f}_{\theta^-}(\hat{\mathbf{x}}_{t_n}^{\phi}, t_n))$

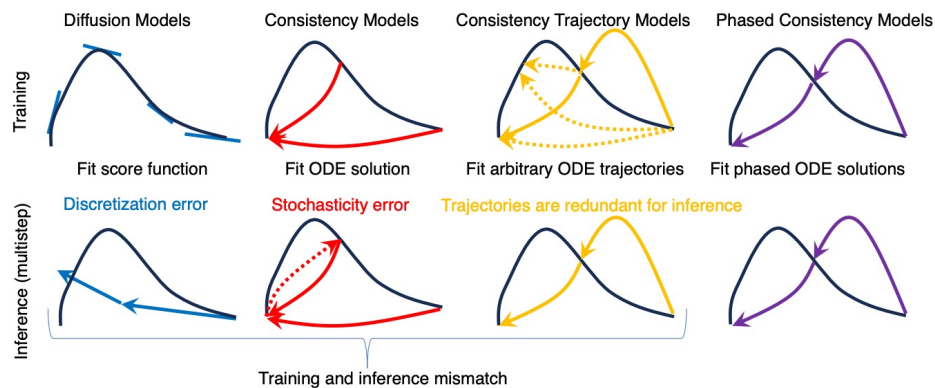
$\theta \leftarrow \theta - \eta \nabla_{\theta} \mathcal{L}(\theta, \theta^-; \phi)$

$\theta^- \leftarrow \text{stopgrad}(\mu \theta^- + (1 - \mu)\theta)$

until convergence



Distillation Techniques: Phased Consistency Distillation



Algorithm 1 Phased Consistency Distillation with CFG-augmented ODE solver (PCD)

Input: dataset \mathcal{D} , initial model parameter θ , learning rate η , ODE solver $\Psi(\cdot, \cdot, \cdot, \cdot)$, distance metric $d(\cdot, \cdot)$, EMA rate μ , noise schedule α_t, σ_t , guidance scale $[w_{\min}, w_{\max}]$, number of ODE step k , discretized timesteps $t_0 = \epsilon < t_1 < t_2 < \dots < t_N = T$, edge timesteps $s_0 = t_0 < s_1 < s_2 < \dots < s_M = t_N \in \{t_i\}_{i=0}^N$ to split the ODE trajectory into M sub-trajectories.

Training data : $\mathcal{D}_x = \{(\mathbf{x}, \mathbf{c})\}$

$\theta^- \leftarrow \theta$

repeat

 Sample $(\mathbf{z}, \mathbf{c}) \sim \mathcal{D}_z, n \sim \mathcal{U}[0, N - k]$ and $\omega \sim [\omega_{\min}, \omega_{\max}]$

 Sample $\mathbf{x}_{t_{n+k}} \sim \mathcal{N}(\alpha_{t_{n+k}} \mathbf{z}; \sigma_{t_{n+k}}^2 \mathbf{I})$

 Determine $[s_m, s_{m+1}]$ given n

$\mathbf{x}_{t_n}^\phi \leftarrow (1 + \omega)\Psi(\mathbf{x}_{t_{n+k}}, t_{n+k}, t_n, \mathbf{c}) - \omega\Psi(\mathbf{x}_{t_{n+k}}, t_{n+k}, t_n, \emptyset)$

$\tilde{\mathbf{x}}_{s_m} = \mathbf{f}_\theta^m(\mathbf{x}_{t_{n+k}}, t_{n+k}, \mathbf{c})$ and $\hat{\mathbf{x}}_{s_m} = \mathbf{f}_{\theta^-}(\mathbf{x}_{t_n}^\phi, t_n, \mathbf{c})$

 Obtain $\tilde{\mathbf{x}}_s$ and $\hat{\mathbf{x}}_s$ through adding noise to $\tilde{\mathbf{x}}_{s_m}$ and $\hat{\mathbf{x}}_{s_m}$

$\mathcal{L}(\theta, \theta^-) = d(\tilde{\mathbf{x}}_{s_m}, \hat{\mathbf{x}}_{s_m}) + \lambda(\text{ReLU}(1 + \tilde{\mathbf{x}}_s) + \text{ReLU}(1 - \hat{\mathbf{x}}_s))$

$\theta \leftarrow \theta - \eta \nabla_\theta \mathcal{L}(\theta, \theta^-)$

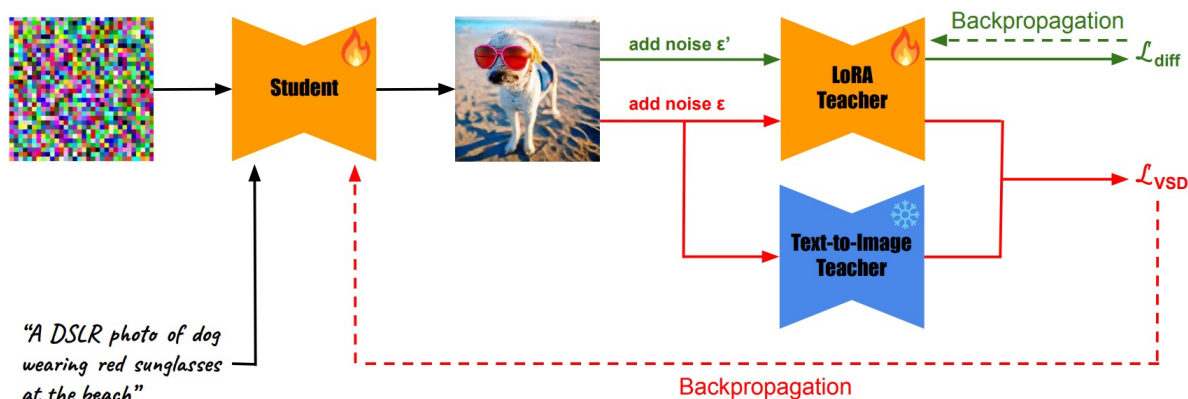
$\theta^- \leftarrow \text{stopgrad}(\mu \theta^- + (1 - \mu)\theta)$

until convergence



Distillation Techniques: Score Distillation

$$\nabla_{\theta} \mathcal{L}_{\text{VSD}}(\theta) \triangleq \mathbb{E}_{t, \epsilon, c} \left[\omega(t) (\epsilon_{\text{pretrain}}(\mathbf{x}_t, t, y^c) - \epsilon_{\phi}(\mathbf{x}_t, t, c, y)) \frac{\partial g(\theta, c)}{\partial \theta} \right]$$

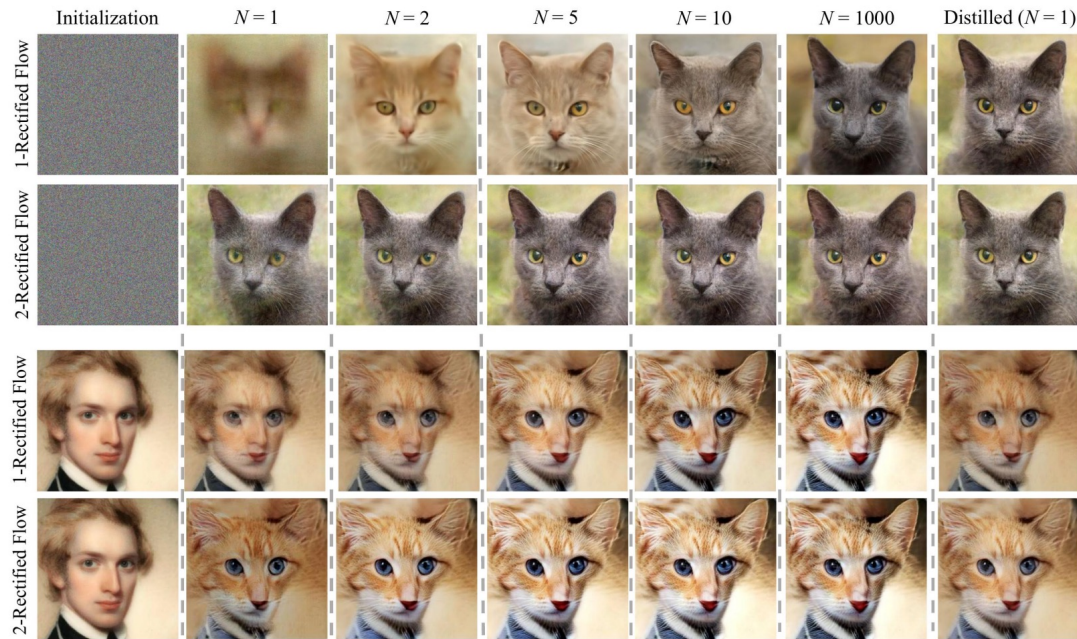


Algorithm 1 SwiftBrush Distillation

- 1: **Require:** a pretrained text-to-image teacher ϵ_{ψ} , a LoRA teacher ϵ_{ϕ} , a student model f_{θ} , two learning rates η_1 and η_2 , a weighting function ω , a prompts dataset Y , the maximum number of time steps T and the noise schedule $\{(\alpha_t, \sigma_t)\}_{t=1}^T$ of the teacher model
- 2: **Initialize:** $\phi \leftarrow \psi, \theta \leftarrow \psi$
- 3: **while not converged do**
- 4: Sample input noise $z \sim \mathcal{N}(0, I)$
- 5: Sample text caption input $y \sim Y$
- 6: Compute student output $\hat{x}_0 = f_{\theta}(z, y)$
- 7: Sample timestep $t \sim \mathcal{U}(0.02T, 0.98T)$
- 8: Sample added noise $\epsilon \sim \mathcal{N}(0, I)$
- 9: Compute noisy sample $\hat{x}_t = \alpha_t \hat{x}_0 + \sigma_t \epsilon$
- 10: $\theta \leftarrow \theta - \eta_1 \left[\omega(t) (\epsilon_{\psi}(\hat{x}_t, t, y) - \epsilon_{\phi}(\hat{x}_t, t, y)) \frac{\partial \hat{x}_0}{\partial \theta} \right]$
- 11: Sample timestep $t' \sim \mathcal{U}(0, T)$
- 12: Sample added noise $\epsilon' \sim \mathcal{N}(0, I)$
- 13: Compute noisy sample $\hat{x}_{t'} = \alpha_{t'} \hat{x}_0 + \sigma_{t'} \epsilon'$
- 14: $\phi \leftarrow \phi - \eta_2 \nabla_{\phi} \|\epsilon_{\phi}(\hat{x}_{t'}, t', y) - \epsilon'\|^2$
- 15: **end while**
- 16: **return** trained student model f_{θ}



Distillation Techniques: Rectified Flow



Advantages:

- High-quality few-step generation.
- Flexibility on inference steps.
- Simple forms.



Distillation Techniques: Rectified Flow

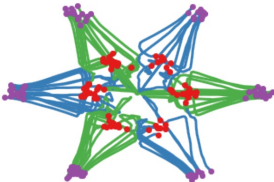
- Linear interpolation.

$$X_t = tX_1 + (1 - t)X_0$$

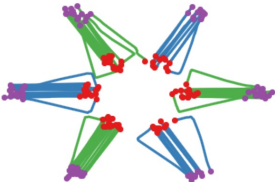
- v -prediction.

$$dX_t = (X_1 - X_0)dt$$

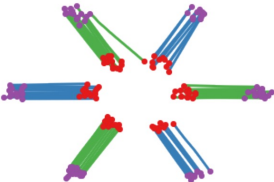
- Rectification (Reflow).



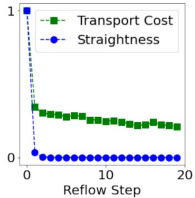
(a) The 1st rectified flow Z^1
 $Z^1 = \text{RectFlow}((X_0, X_1))$



(b) Reflow Z^2
 $Z^2 = \text{RectFlow}((Z_0^1, Z_1^1))$



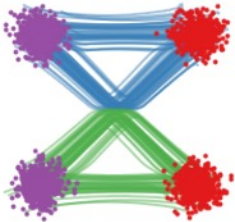
(c) Reflow Z^3
 $Z^3 = \text{RectFlow}((Z_0^2, Z_1^2))$



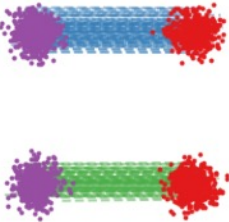
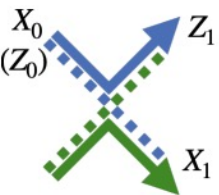
(d) Transport cost, Straightness



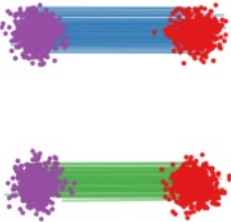
(a) Linear interpolation
 $X_t = tX_1 + (1 - t)X_0$



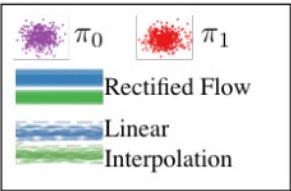
(b) Rectified flow Z_t
 induced by (X_0, X_1)



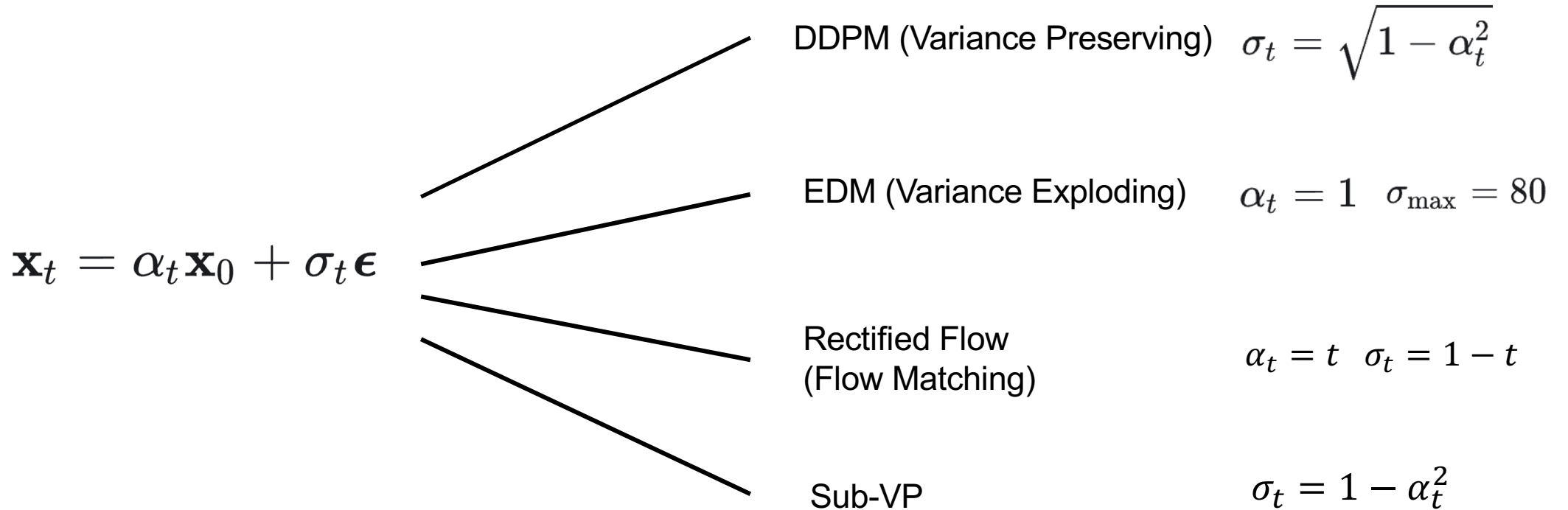
(c) Linear interpolation
 $Z_t = tZ_1 + (1 - t)Z_0$



(d) Rectified flow Z'_t
 induced by (Z_0, Z_1)



Diffusion Models: A (relative) Unified Perspective



The Magic of Rectified Flow: Retraining with Matched Noise-Sample Pairs

Algorithm 1 Flow Matching v -Prediction

Input:

Sample \mathbf{x}_0 from the data distribution

Sample time t from a predefined schedule or uniformly from $[0, 1]$

Sample noise ϵ from normal distribution

Compute \mathbf{x}_t : $\mathbf{x}_t = (1 - t) \cdot \mathbf{x}_0 + t \cdot \epsilon$

Predict velocity \hat{v} using the model: $\hat{v} = \text{Model}(\mathbf{x}_t, t)$

Compute loss: $\mathcal{L} = \|\hat{v} - (\mathbf{x}_0 - \epsilon)\|_2^2$

Backpropagate and update parameters

Algorithm 3 Rectified Flow v -Prediction

Input: noise-data pair $(\epsilon, \hat{\mathbf{x}}_0)$

~~Sample \mathbf{x}_0 from the data distribution~~

Sample time t from a predefined schedule or uniformly from $[0, 1]$

~~Sample noise ϵ from normal distribution~~

Compute \mathbf{x}_t : $\mathbf{x}_t = (1 - t) \cdot \hat{\mathbf{x}}_0 + t \cdot \epsilon$

Predict velocity \hat{v} using the model: $\hat{v} = \text{Model}(\mathbf{x}_t, t)$

Compute loss: $\mathcal{L} = \|\hat{v} - (\hat{\mathbf{x}}_0 - \epsilon)\|_2^2$

Backpropagate and update parameters



Flow Matching Training Is a Subset of Diffusion Training

Algorithm 1 Flow Matching v -Prediction

Input:

Sample \mathbf{x}_0 from the data distribution

Sample time t from a predefined schedule or uniformly from $[0, 1]$

Sample noise ϵ from normal distribution

Compute \mathbf{x}_t : $\mathbf{x}_t = (1 - t) \cdot \mathbf{x}_0 + t \cdot \epsilon$

Predict velocity \hat{v} using the model: $\hat{v} = \text{Model}(\mathbf{x}_t, t)$

Compute loss: $\mathcal{L} = \|\hat{v} - (\mathbf{x}_0 - \epsilon)\|_2^2$

Backpropagate and update parameters

Algorithm 2 Diffusion Training ϵ -Prediction

Input: α_t, σ_t

Sample \mathbf{x}_0 from the data distribution

Sample time t from a predefined schedule or uniformly from $[0, 1]$

Sample noise ϵ from normal distribution

Compute \mathbf{x}_t : $\mathbf{x}_t = \alpha_t \cdot \mathbf{x}_0 + \sigma_t \cdot \epsilon$

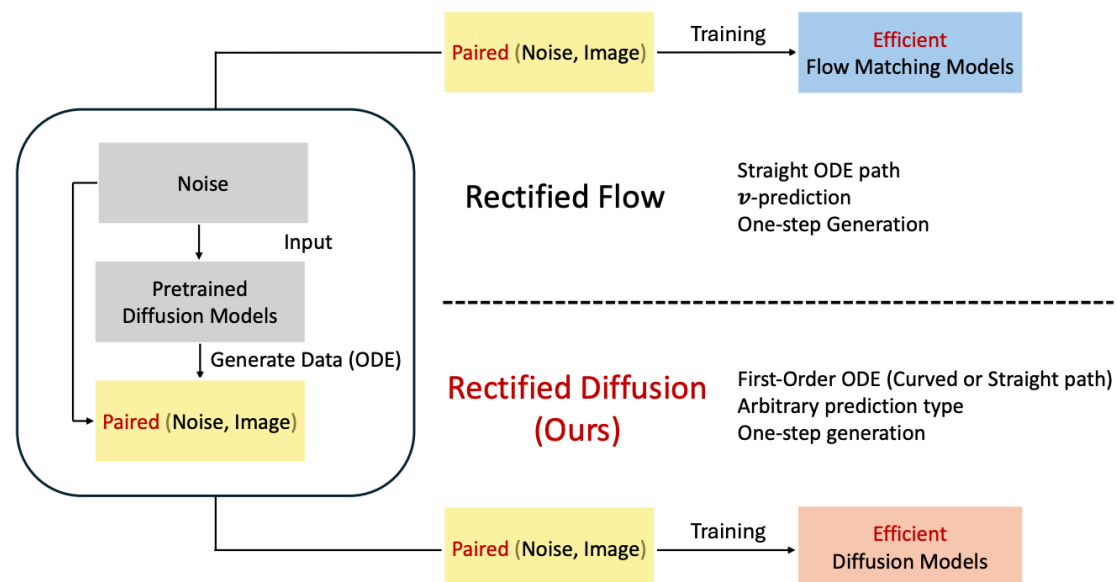
Predict noise $\hat{\epsilon}$ using the model: $\hat{\epsilon} = \text{Model}(\mathbf{x}_t, t)$

Compute loss: $\mathcal{L} = \|\hat{\epsilon} - \epsilon\|_2^2$

Backpropagate and update parameters



Rectified Diffusion: Extending Rectified Flow to General Diffusion Models



Algorithm 4 Rectified Diffusion ϵ -Prediction

Input: noise-data pair $(\epsilon, \hat{\mathbf{x}}_0)$, α_t, σ_t

Sample \mathbf{x}_0 from the data distribution

Sample time t from a predefined schedule or uniformly from $[0, 1]$

Sample noise ϵ from normal distribution

Compute \mathbf{x}_t : $\mathbf{x}_t = \alpha_t \cdot \hat{\mathbf{x}}_0 + \sigma_t \cdot \epsilon$

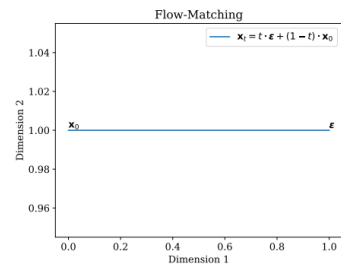
Predict noise $\hat{\epsilon}$ using the model: $\hat{\epsilon} = \text{Model}(\mathbf{x}_t, t)$

Compute loss: $\mathcal{L} = \|\hat{\epsilon} - \epsilon\|_2^2$

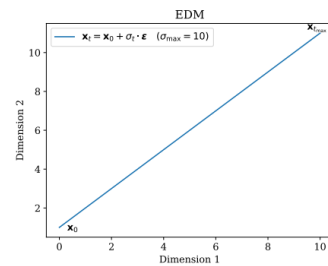
Backpropagate and update parameters



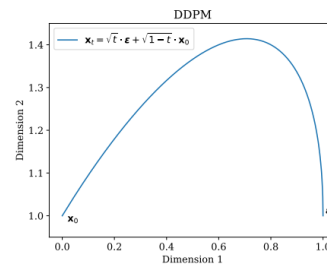
Rectified Diffusion: the Essential Training Target Is First-Order ODE



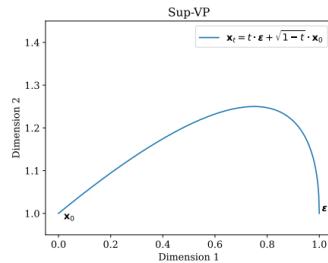
(a) Flow Matching



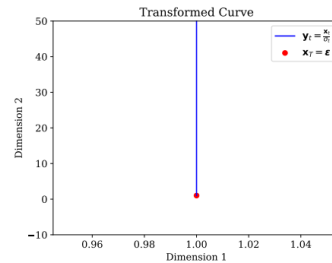
(b) EDM



(c) DDPM



(d) Sub-VP



(e) Transformed

Important Points of First-Order ODE:

- It has the same form of predefined diffusion forms.

$$\mathbf{x}_t = \alpha_t \mathbf{x}_0 + \sigma_t \epsilon$$

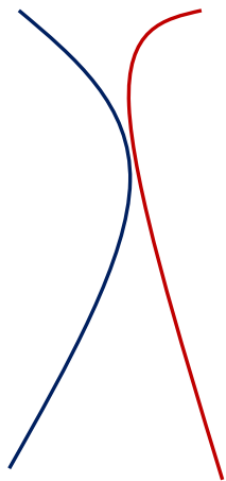
- It can be inherently curved.

- It can be transformed into straight lines with timestep dependent scaling.

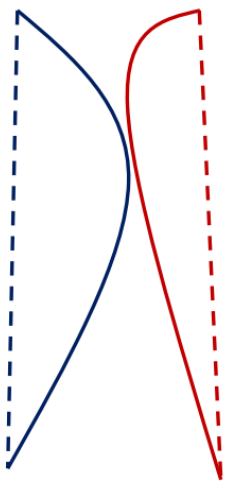
$$\mathbf{y}_t = \frac{\alpha_t}{\sigma_t} \mathbf{x}_0 + \epsilon$$



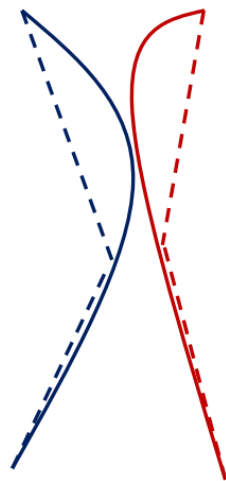
Rectified Diffusion (Phased)



ODE trajectory of
Pretrained Diffusion Models



ODE trajectory of
Rectified Diffusion Models

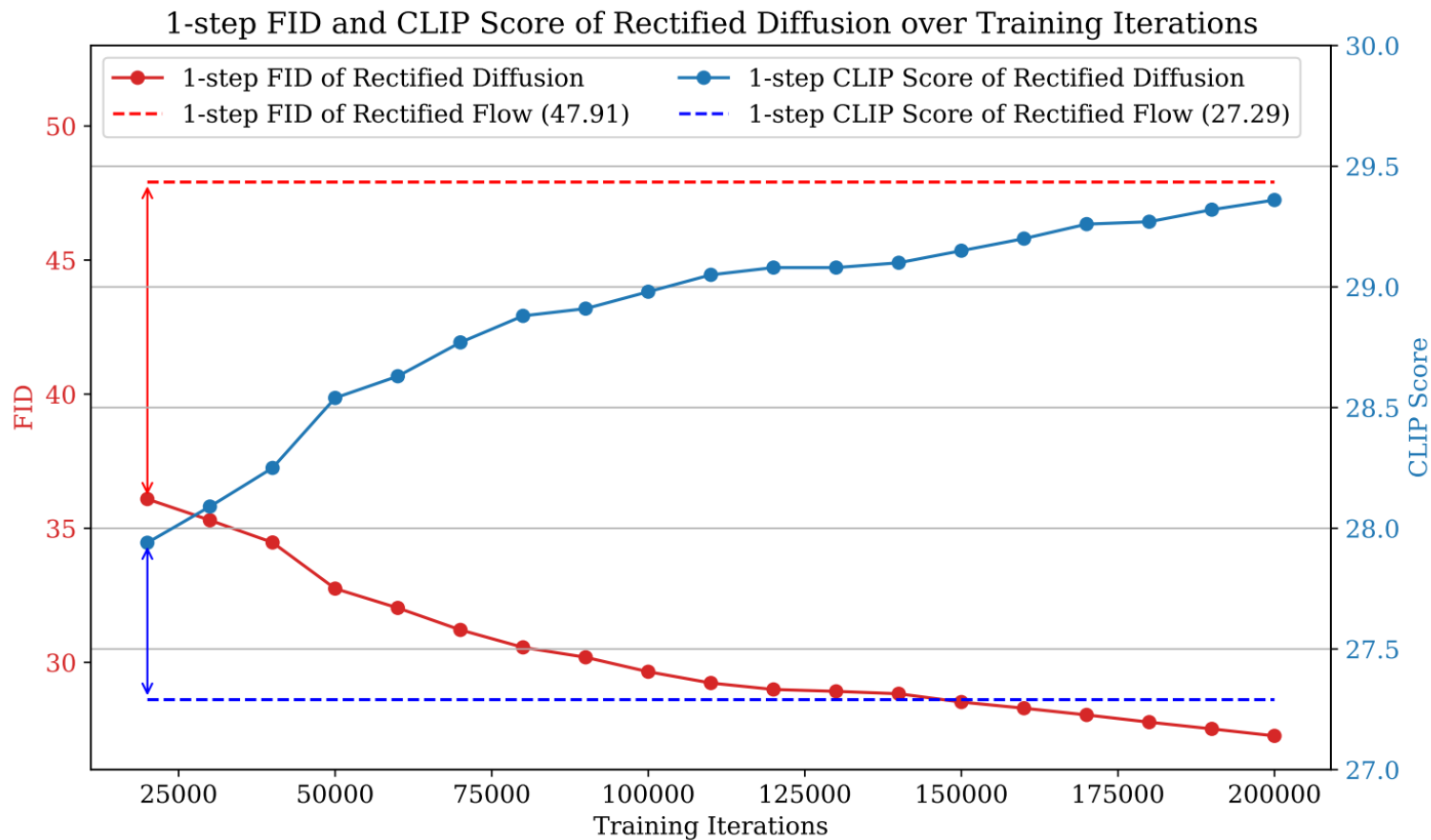


ODE trajectory of
Phased Rectified Diffusion Models

$$\epsilon = \frac{\frac{\mathbf{x}_{s_{m-1}}}{\alpha_{s_{m-1}}} - \frac{\mathbf{x}_{s_m}}{\alpha_{s_m}}}{\frac{\sigma_{s_{m-1}}}{\alpha_{s_{m-1}}} - \frac{\sigma_{s_m}}{\alpha_{s_m}}} = \frac{\Delta \mathbf{z}}{\Delta \text{NSR}}$$



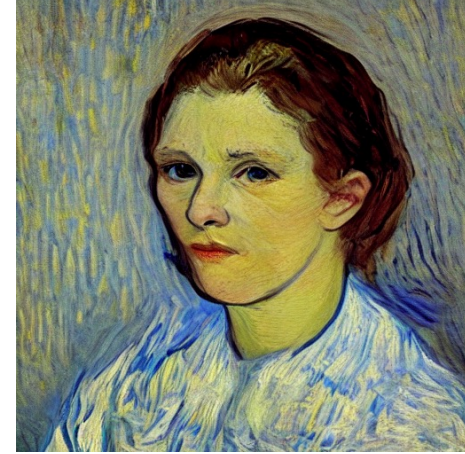
Rectified Diffusion Vs Rectified Flow



Rectified Diffusion Vs Rectified Flow



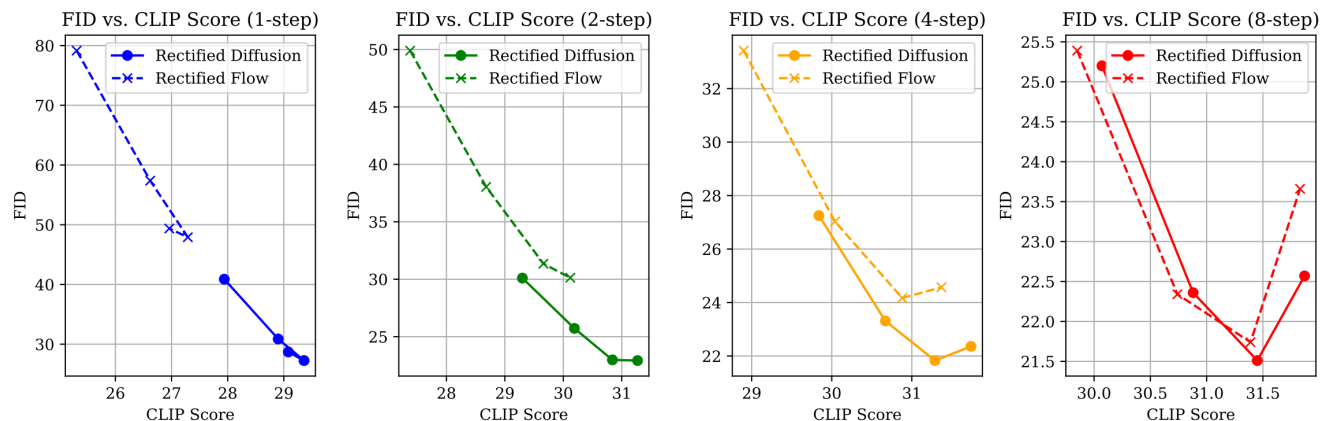
Rectified-Flow



Rectified-Diffusion



Rectified Diffusion Vs Rectified Flow



Method	Res.	Time (↓)	# Steps	# Param.	FID (↓)	CLIP (↑)
SDv1-5+DPMSolver (Upper-Bound) (Lu et al., 2022)	512	0.88s	25	0.9B	20.1	0.318
Rectified Flow (Liu et al., 2023)	512	0.88s	25	0.9B	21.65	0.315
Rectified Flow (Liu et al., 2023)	512	0.09s	1	0.9B	47.91	0.272
Rectified Flow (Liu et al., 2023)	512	0.13s	2	0.9B	31.35	0.296
Rectified Diffusion (Ours)	512	0.09s	25	0.9B	21.28	0.316
Rectified Diffusion (Ours)	512	0.09s	1	0.9B	27.26	0.295
Rectified Diffusion (Ours)	512	0.13s	2	0.9B	22.98	0.309
Rectified Flow (Distill) (Liu et al., 2023)	512	0.09s	1	0.9B	23.72	0.302
Rectified Flow (Distill) (Liu et al., 2023)	512	0.13s	2	0.9B	73.49	0.261
Rectified Flow (Distill) (Liu et al., 2023)	512	0.21s	4	0.9B	103.48	0.245
Rectified Diffusion (CD) (Ours)	512	0.09s	1	0.9B	22.83	0.305
Rectified Diffusion (CD) (Ours)	512	0.13s	2	0.9B	21.66	0.312
Rectified Diffusion (CD) (Ours)	512	0.21s	4	0.9B	21.43	0.314
PeRFlow (Yan et al., 2024)	512	0.21s	4	0.9B	22.97	0.294
Rectified Diffusion (Phased) (Ours)	512	0.21s	4	0.9B	20.64	0.311
PeRFlow-SDXL (Yan et al., 2024)	1024	0.71s	4	3B	27.06	0.335
Rectified Diffusion-SDXL (Phased) (Ours)	1024	0.71s	4	3B	25.81	0.341



References

- [1] Denoising Diffusion Probabilistic Models.
- [2] Denoising Diffusion Implicit Models.
- [3] Score-Based Generative Modeling through Stochastic Differential Equations.
- [4] Flow Matching for Generative Modeling.
- [5] Elucidating the Design Space of Diffusion-Based Generative Models.
- [6] DPM-Solver: A Fast ODE Solver for Diffusion Probabilistic Model Sampling in Around 10 Steps.
- [7] Discrete Flow Matching.
- [8] Consistency Models.
- [9] Consistency Models Made Easy.
- [10] Latent Consistency Models: Synthesizing High-Resolution Images with Few-step Inference.
- [11] Phased Consistency Model.
- [12] Multistep Consistency Models.
- [13] PeRFlow: Piecewise Rectified Flow as Universal Plug-and-Play Accelerator.
- [14] Flow Straight and Fast: Learning to Generate and Transfer Data with Rectified Flow.
- [15] InstaFlow: One Step is Enough for High-Quality Diffusion-Based Text-to-Image Generation.
- [16] StyleGAN-T: Unlocking the Power of GANs for Fast Large-Scale Text-to-Image Synthesis.
- [17] Stable Consistency Tuning: Understanding and Improving Consistency Models.
- [18] SwiftBrush : One-Step Text-to-Image Diffusion Model with Variational Score Distillation.
- [19] One-step Diffusion with Distribution Matching Distillation.
- [20] Progressive Distillation for Fast Sampling of Diffusion Models



Our Works

[1] Stable Consistency Tuning: Understanding and Improving Consistency Models.

[2] Rectified Diffusion: Straightness Is Not Your Need in Rectified Flow.

[3] Phased Consistency Model.

[4] AnimateLCM: Computation-Efficient Personalized Style Video Generation without Personalized Video Data.



Thank you!

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